4 LINEAR EQUATIONS IN TWO VARIABLES

EXERCISE 4.1

Q.1. *The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this statement.*
(Take the cost of a notebook to be Rs x and that of a pen to be Rs y.)

Sol. Cost of a notebook = x
Cost of a pen = y
Then according to given statement
$$x = 2y \quad \text{or} \quad x - 2y = 0$$

Q.2. *Express the following linear equations in the form ax + by + c = 0 and indicate the values of a, b and c in each case :*

(i) $2x + 3y = 9.3\overline{5}$ (ii) $x - \frac{y}{5} - 10 = 0$ (iii) $-2x + 3y = 6$
(iv) $x = 3y$ (v) $2x = -5y$ (vi) $3x + 2 = 0$
(vii) $y - 2 = 0$ (viii) $5 = 2x$

Sol. (i) $2x + 3y = 9.3\overline{5}$
$\Rightarrow 2x + 3y - 9.3\overline{5} = 0$
So, $a = 2,\ b = 3,\ c = -9.3\overline{5}$

(ii) $x - \frac{y}{5} - 10 = 0$
So, $a = 1,\ b = \frac{-1}{5},\ c = -10$

(iii) $-2x + 3y = 6$
$\Rightarrow -2x + 3y - 6 = 0$
So, $a = -2,\ b = 3,\ c = -6$

(iv) $x = 3y \Rightarrow x - 3y + 0 = 0$
So, $a = 1,\ b = -3,\ c = 0$

(v) $2x = -5y$
$\Rightarrow 2x + 5y + 0 = 0$
$a = 2,\ b = 5,\ c = 0$

(vi) $3x + 2 = 0$
$\Rightarrow 3x + 0 \cdot y + 2 = 0$
So, $a = 3,\ b = 0,\ c = 2$

(vii) $y - 2 = 0$
$\Rightarrow 0 \cdot x + 1 \cdot y - 2 = 0$
So, $a = 0,\ b = 1,\ c = -2$

(viii) $5 = 2x$
$\Rightarrow 5 - 2x = 0$
$\Rightarrow -2x + 0 \cdot y + 5 = 0$
So, $a = -2,\ b = 0,\ c = 5$

4 LINEAR EQUATIONS IN TWO VARIABLES

EXERCISE 4.2

Q.1. *Which one of the following options is true, and why?*

$y = 3x + 5$ has

(i) a unique solution, (ii) only two solutions

(iii) infinitely many solutions

Sol. (iii) infinitely many solutions. It is because a linear equation in two variables has infinitely many solutions. We keep changing the value of x and solve the linear equation for the corresponding value of y.

Q.2. *Write four solutions for each of the following equations :*

(i) $2x + y = 7$ (ii) $\pi x + y = 9$ (iii) $x = 4y$

Sol. (i) $2x + y = 7$

Let $x = 1$

Then, $2 \times 1 + y = 7$

$\Rightarrow y = 7 - 2 = 5.$

$\therefore$ (1, 5) is a solution.

Let $x = 2$

Then, $2 \times 2 + y = 7$

$\Rightarrow y = y - 4 = 3$

$\Rightarrow$ (2, 3) is another solution.

Let $x = 3$

Then, $2 \times 3 + y = 7$

$\Rightarrow y = 7 - 6 = 1$

$\Rightarrow$ (3, 1) is another solution.

Let $x = 4$

Then, $2 \times 8 + y = 7$

$\Rightarrow y = -8 + 7 = -1$ $\Rightarrow$ (4, –1) is another solution.

Therefore, (1, 5), (2, 3), (3, 1) and (4, –1) are all solutions of $2x + y = 7$

(ii) $\pi x + y = 9$

Let $x = \frac{1}{\pi}$,

Then, $\pi \times \frac{1}{\pi} + y = 9$

$\Rightarrow y = 9 - 1 = 8.$ $\therefore \left(\frac{1}{\pi}, 8\right)$ is a solution.

Let $x = \frac{2}{\pi}$

Then, $\pi \times \frac{2}{\pi} + y = 9$

$\Rightarrow y = 9 - 2 = 7.$ $\therefore \left(\frac{2}{\pi}, 7\right)$ is a solution.

Let $x = \frac{3}{\pi}$

Then, $\pi \times \frac{3}{\pi} + y = 9$

$\Rightarrow y = 9 - 3 = 6.$ $\therefore \left(\frac{3}{\pi}, 6\right)$ is a solution.

Let $x = \frac{4}{\pi}$

Then, $\pi \times \frac{4}{\pi} + y = 9$

$\Rightarrow y = 5.$ $\therefore \left(\frac{4}{\pi}, 5\right)$ is a solution.

Therefore, $\left(\frac{1}{\pi}, 8\right)$, $\left(\frac{2}{\pi}, 7\right)$, $\left(\frac{3}{\pi}, 6\right)$ and $\left(\frac{4}{\pi}, 5\right)$ are all solutions of the equation $\pi x + y = 9$.

(iii) $x = 4y$

Let $x = 8$

Then, $8 = 4y \Rightarrow y = 2.$ $\therefore$ (8, 2) is a solution.

Let $x = 12$

Then, $12 = 4y, \Rightarrow y = 3.$ $\therefore$ (12, 3) is a solution.

Let $x = 16$

Then, $16 = 4y, \Rightarrow y = 4.$ $\therefore$ (16, 4) is a solution.

Let $x = 20$

Then, $20 = 4y, \Rightarrow y = 5.$ $\therefore$ (20, 5) is a solution.

Therefore, (8, 2), (12, 3), (16, 4) and (20, 5) are all solutions of $x = 4y$.

Q.3. *Check which of the following are solutions of the equation $x - 2y = 4$ and which are not :*

(i) (0, 2) *(ii)* (2, 0) *(iii)* (4, 0)

(iv) $(\sqrt{2}, 4\sqrt{2})$ *(v)* (1, 1)

Sol. $x - 2y = 4$

(i) When $x = 0, y = 2$, then

$0 - 4 = 4. \Rightarrow$ R.H.S $\neq$ L.H.S. Therefore, it is not a solution.

(ii) When $x = 2, y = 0$

$2 - 0 = 4. \Rightarrow$ R.H.S $\neq$ L.H.S. Therefore, (2, 0) is not a solution

(iii) When $x = 4, y = 0$

$4 - 2 \times 0 = 4.$ L.H.S = R.H.S. Therefore, (4, 0) is a solution.

(iv) When $x = \sqrt{2}$, $y = 4\sqrt{2}$

$\sqrt{2} - 8\sqrt{2} = 4.$ L.H.S $\neq$ R.H.S.

Therefore $(\sqrt{2}, 4\sqrt{2})$ is not a solution

(v) When $x = 1, y = 1, 1 - 2 \times 1 = 4.$ L.H.S $\neq$ R.H.S.

Therefore (1,1) is not a solution

Q.4. *Find the value of k, if $x = 2, y = 1$ is a solution of the equation $2x + 3y = k$.*

Sol. $x = 2, y = 1$ is a solution of $2x + 3y = k$.

Thus, $2 \times 2 + 3 \times 1 = k$

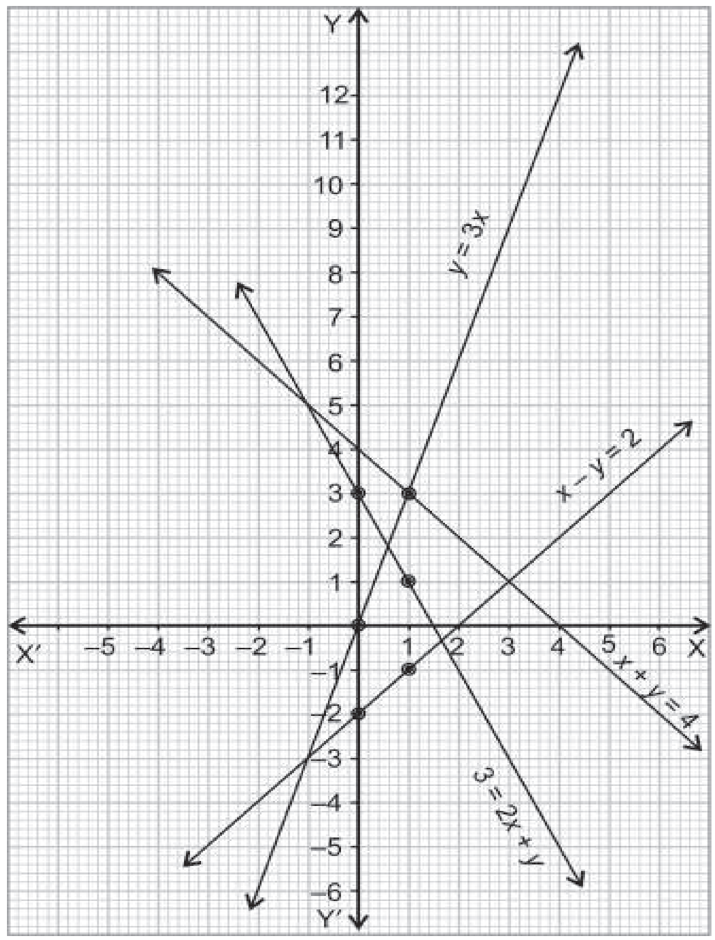
$\Rightarrow$ $4 + 3 = 7 = k.$ $\Rightarrow k = 7$

4 LINEAR EQUATIONS IN TWO VARIABLES

EXERCISE 4.3

Q.1. *Draw the graph of each of the following linear equations in two variables :*
(i) $x + y = 4$ *(ii)* $x - y = 2$ *(iii)* $y = 3x$ *(iv)* $3 = 2x + y$

Sol. (i) $x + y = 4$
For $x = 0$, $y = 4$
For $x = 1$, $y = 3$;
(ii) $x - y = 2$
For $x = 0$, $y = -2$
For $x = 1$, $y = -1$
So, we have the points $(0, -2)$ and $(1, -1)$.
(iii) $y = 3x$
For $x = 0$, $y = 0$
For $x = 1$, $y = 3$
So, we have the points $(0, -2)$ and $(1, -1)$.
(iv) $3 = 2x + y$
For $x = 0$, $y = 3$
$x = 1$, $y = 1$
So, we have the points $(0, 3)$ and $(1, 1)$.



Q.2. *Give the equations of two lines passing through* (2, 14). *How many more such lines are there, and why?*

Sol. Let $x + y = k$ be such a line,
then, $2 + 14 = k$, $k = 16$.
$\therefore$ $x + y = 16$ passes through (2, 14).
Let $2x + 3y = k'$ be another line through (2, 14).
$2 \times 2 + 3 \times 14 = k'$
$\Rightarrow k' = \; = 4 + 42 = 46$
$\Rightarrow 2x + 3y = 46$ passes through (2, 14).
There are infinitely many such lines, as through a point infinite number of straight lines can be drawn.

Q.3. *If the point (3, 4) lies on the graph of the equation 3y = ax + 7, find the value of a.*

Sol. (3, 4) lies on $3y = ax + 7$

Therefore, substituting 3 for x and 4 for y in the above equation, we have

$$3 \times 4 = a \times 3 + 7$$

$$\Rightarrow 3a + 7 = 12$$

$$\Rightarrow 3a = 5$$

$$\Rightarrow \quad a = \frac{5}{3}$$

Q.4. *The taxi fare in a city is as follows : For the first kilometre, the fare is Rs 8 and for the subsequent distance it is Rs 5 per km. Taking the distance covered as x km and total fare as Rs y, write a linear equation for this information, and draw its graph.*

Sol. Total fare

$y = 1 \times 8 + (x - 1)\,5$,

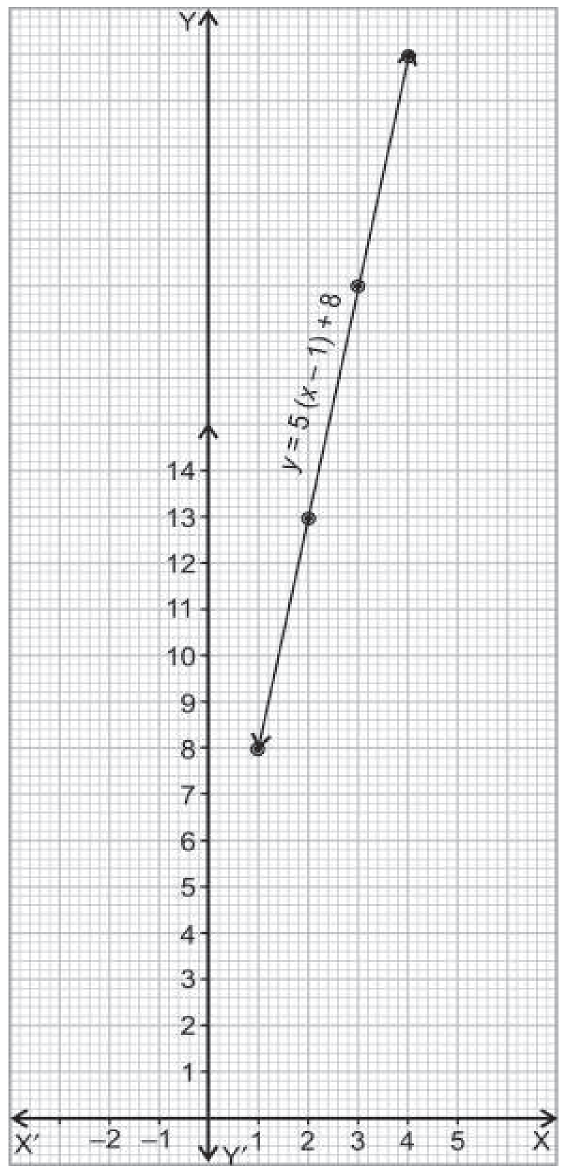
where x is in km and y in Rs.

So, the required equation is $y = 5(x - 1) + 8$

Now, for $x = 1$

$y = 2$, $y = 5 + 8 = 13$

So, the points are (1, 8) and (2, 13)



Q.5. *From the choices given below, choose the equation whose graphs are given in Fig. A and Fig. B.*

For Fig. A

(i) $y = x$

(ii) $x + y = 0$

(iii) $y = 2x$

(iv) $2 + 3y = 7x$

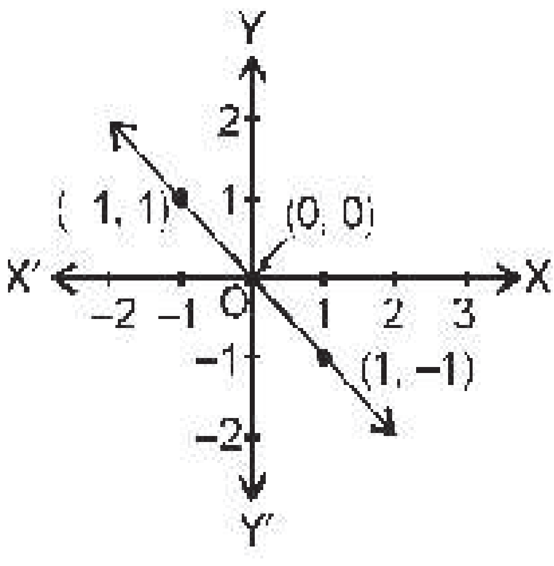
For Fig. B

(i) $y = x + 2$

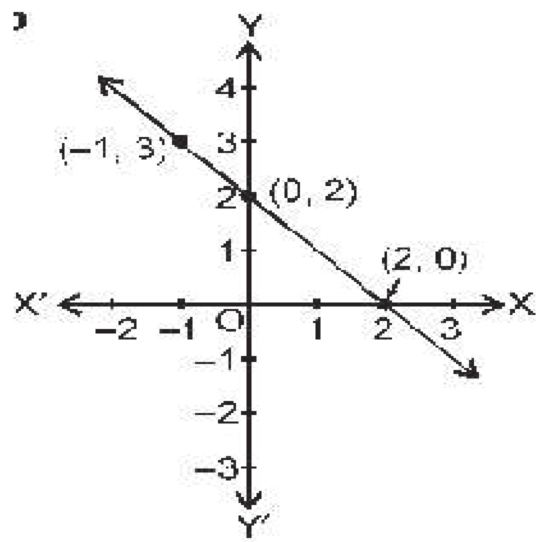
(ii) $y = x - 2$

(iii) $y = -x + 2$

(iv) $x + 2y = 6$



A



B

Sol. For fig. A

(ii) $x + y = 0$. This can be verified by putting $x = 1, y = -1$ in $x + y = 0$

For fig. B

(iii) $y = -x + 2$ is the right equation.

This can be verified by putting $x = -1, y = 3$ in $y = -x + 2$

Q.6. *If the work done by a body on application of a constant force is directly proportional to the distance travelled by the body, express this in the form of an equation in two variables and draw the graph of the same by taking the constant force as 5 units. Also read from the graph the work done when the distance travelled by the body is :*

(i) 2 units (ii) 0 units.

Sol. Let y = work done,

F = constant force = 5,

x = distance travelled,

then $y = 5x$.

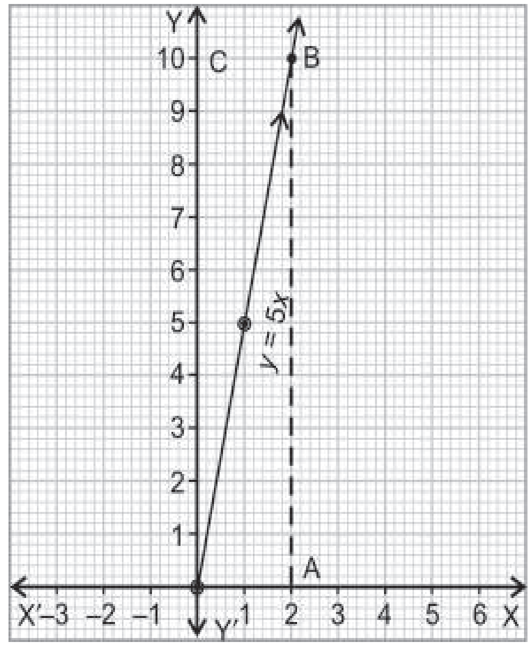
For, $x = 0$, $y = 0$, for $x = 1$, $y = 5$, for $x = 2$, $y = 10$.

$x = 3$, $y = 15$. $x = 4$, $y = 20$.

(i) Let A represent $x = 2$. The x-axis. From A, draw AB ⊥ x-axis, meeting the graph at B. From B, draw BC ⊥ y-axis, meeting y - axis at C. The ordinate of c is 10.

So, when the distance travelled is 24 units. The work done.

(ii) For $x = 0$, $y = 0$. From the graph.



Q.7. *Yamini and Fatima, two students of Class IX of a school, together contributed Rs 100 towards the Prime Minister's Relief Fund to help the earthquake victims. Write a linear equation which satisfies this data. (You may take their contributions as Rs x and Rs y.) Draw the graph of the same.*

Sol. $x + y = 100$ is the linear equation satisfying the given data.

For, $x = 0$, $y = 100$

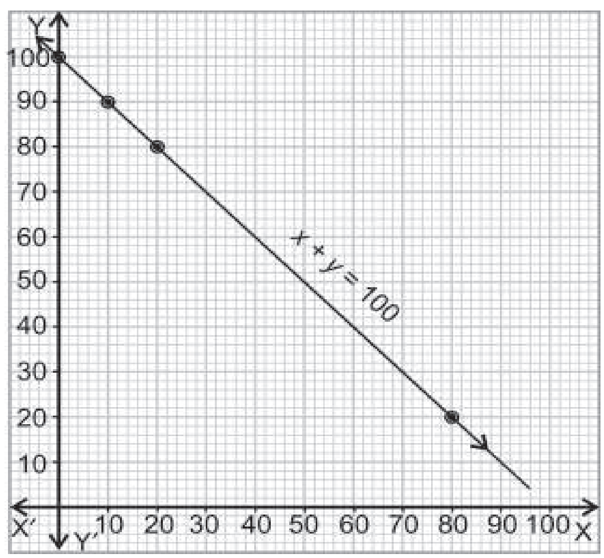
$x = 10$, $y = 90$

$x = 20$, $y = 80$

So, the point are

(0, 100), (10, 90)

and (20, 80)



Q.8. *In countries like USA and Canada, temperature is measured in Fahrenheit, whereas in countries like India, it is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius :*

$$F = \left(\frac{9}{5}\right)C + 32$$

(i) *Draw the graph of the linear equation above using Celsius for x-axis and Fahrenheit for y-axis.*

(ii) *If the temperature is 30°C, what is the temperature in Fahrenheit?*

(iii) *If the temperature is 95°F, what is the temperature in Celsius?*

(iv) *If the temperature is 0°C, what is the temperature in Fahrenheit and if the temperature is 0°F, what is the temperature in Celsius?*

(v) *Is there a temperature which is numerically the same in both Fahrenheit and Celsius? If yes, find it.*

Sol. $F = \left(\frac{9}{5}\right)C + 32$

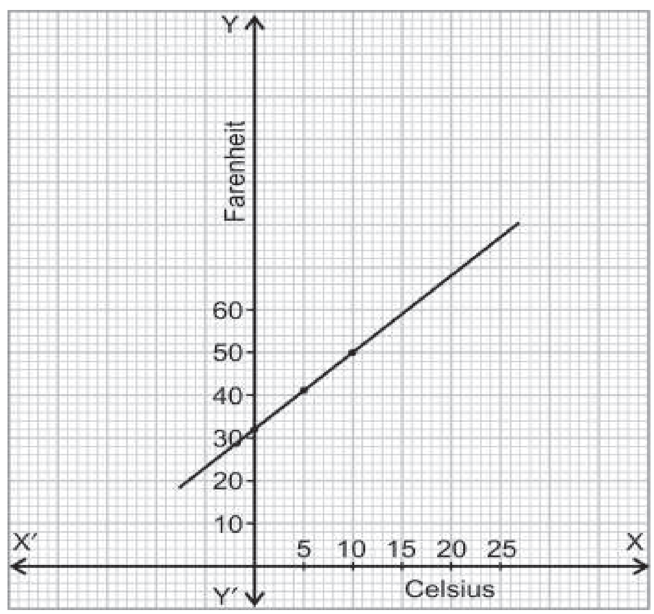
(i) For, C = 0, F = 32

For, C = 5, F = 41

For, C = 10, F = 50

(ii) For 30°C, corres-ponding Fahrenheit temperature is 86°F.

(iii) For F = 95, $F = 95 = \left(\frac{9}{5}\right)C + 32$



$\Rightarrow \left(\frac{9}{5}\right) C = 95 - 32$

$= 63$

$\Rightarrow C = \frac{5}{9} \times 63$

$= 35°C$

(iv) For C = 0°

$F = \left(\frac{9}{5}\right) \times 0 + 32 = 32°F.$

For F = 0

$0 = \left(\frac{9}{5}\right) C + 32$

$\Rightarrow C = \frac{-32\times5}{9} = \frac{-160}{9} = -17.8°C.$

(v) Let F = x, C = x.

Then, $x = \left(\frac{9}{5}\right) x + 32$

$$\Rightarrow \quad x - \frac{9}{5}x = 32$$

$$\Rightarrow \quad x\left(1-\frac{9}{5}\right)=32$$

$$\Rightarrow \quad x\left(\frac{-4}{5}\right)=32$$

$$\Rightarrow x = \frac{-32\times5}{4} = -\,40°C$$

Thus –40°C. and – 40°F are equal temperatures.

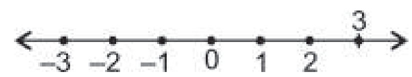
4 LINEAR EQUATIONS IN TWO VARIABLES

EXERCISE 4.4

Q.1. *Give the geometric representations of $y = 3$ as an equation*
(i) in one variable *(ii) in two variables*

Sol. $y = 3$

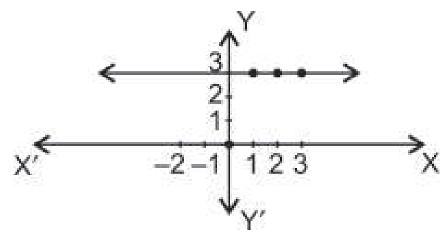
(i) As an equation in one variable, it is the number 3 on the number line.



(ii) As an equation in two variables, it can be written as $0 \cdot x + y = 3$.

Value of x can be any number but of y will continue to be 3.

It is a line parallel to x-axis and 3 units above it.

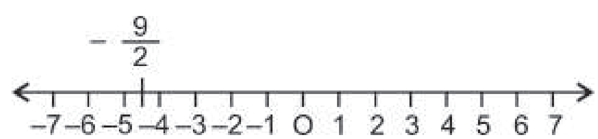


Q.2. *Give the geometric representations of $2x + 9 = 0$ as an equation*
(i) in one variable *(ii) in two variables*

Sol. $2x + 9 = 0$

(i) As an equation in one variable.

$$2x = -9 \Rightarrow x = \frac{-9}{2}$$



(ii) $2x + 0 \cdot y + 9 = 0$ is an equation in two variables. Value of y can be any number but x remains $\frac{-9}{2}$.

It is a line parallel to y-axis and $\frac{9}{2}$ units to the left of O.